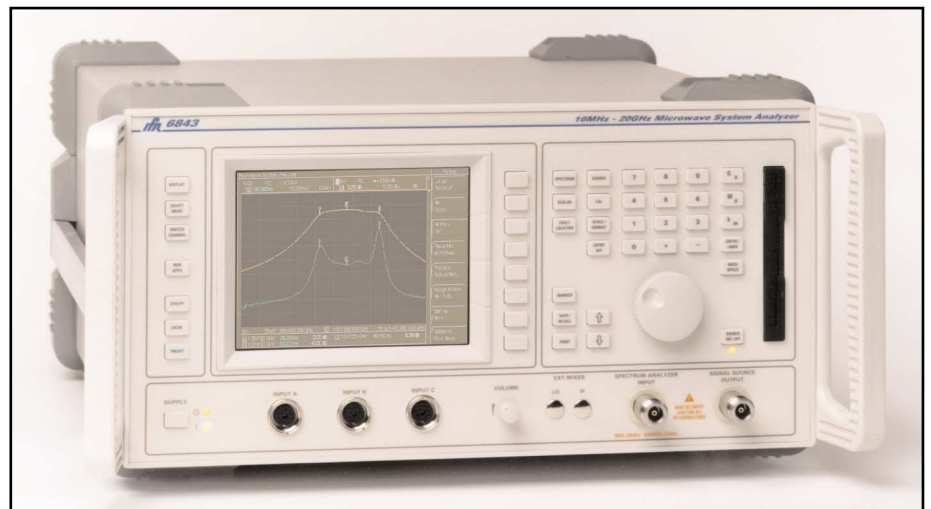# application note

Measurement of Group Delay using the 6840 series Microwave System Analyzer with Option 22

by Adrian Jones and Jason McManus

Measurement of Group Delay and Amplitude response of RF and Microwave components and assemblies, including frequency converters, using the 6840 series Microwave System Analyzer with the Group Delay option.

INTRODUCTION

Who cares about Group Delay?

Anyone who is passing information (i.e. modulated signals) through components, assemblies or networks, or anyone who is manufacturing or testing such components, assemblies or networks. Filters, amplifiers, frequency converters and front ends of radio links are particularly important for group delay.

What is Group Delay?

Group delay is defined as the negative of the derivative (rate of change) of a component or network's phase/frequency response. It is a measure of phase linearity and is defined by the following equation:

$$T_g = - \frac{\partial \phi}{\partial \omega}$$

Equation 1

where ϕ is phase (radians) and ω is angular frequency (radians / second).

Ideal networks pass modulated signals without introducing any distortion. For this they need flat amplitude response and linear (constant slope) phase response over the bandwidth of the signal. Therefore a flat group delay response is needed. All of the frequency components of the signals passing through the component or network will experience the same amount of time delay.

In the real world, however, components and networks are not ideal and will cause distortion through amplitude and phase non-linearities. This can result in a signal that no longer matches the receiver filter, resulting in degraded Signal to Noise Ratio (SNR) or Bit Error Rate (BER).

Flat group delay versus frequency over a given passband implies linear phase and in many cases only group delay flatness is specified. There are, however, situations where absolute group delay is of interest such as equalising the delay of two channels, characterizing cables and the design of quadrature FM demodulators. The 6840 series group delay option can measure both absolute and relative group delay and will display amplitude and group delay response simultaneously. Frequency converting devices can be characterized without access to the internal local oscillator.

A tutorial explanation of group delay, how the 6840 series makes the measurement, frequency modulation and mismatch errors is given in the Appendices.

Conventions

The following conventions are used to indicate keypresses on the MSA.

[**BOLD**]- Hardkey press, i.e. a dedicated front panel function key
[normal] - data entry via numeric keypad
[*Italic*]- Softkey press i.e. software menu key
[●] - toggle function enabled
[○] - toggle function disabled

MAKING MEASUREMENTS

Quick start

This section describes basic operation of the 6840 series group delay measurement facility through an example. In this particular example, the frequency response and group delay of a bandpass filter will be simultaneously displayed between 1.3 and 1.4 GHz.

A scalar analyzer doesn't correct for errors caused by reflections. Good match at the DUT input and output are therefore important. Attenuator pads can be used to improve the matching at the expense of amplitude dynamic range and delay noise (see Appendix 4).

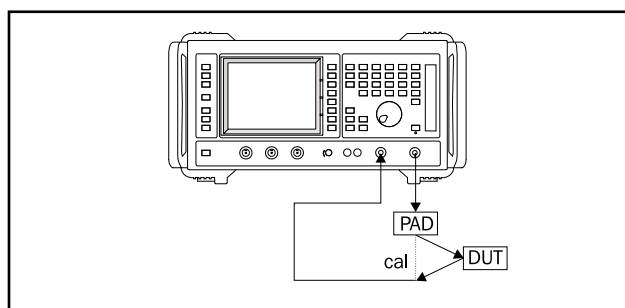Connect the 684x for calibration as shown in Figure 1. A 6 dB pad is suggested.

Figure 1 - Connections for group delay measurement

To set the group delay measurement on measurement 1:-

[**PRESET**] [*Full*]
[**SCALAR**] [*Yes*]
[*Input selection*] [*Group Delay*] [*Group Delay*]
[**SOURCE**] [*Set Start Frequency*] [1.3] [**Gn**]
[*Set Stop Frequency*] [1.4] [**Gn**]
[*Set Output Power*] [6] [**ENTER**]

To set the frequency response measurement to measurement 2:-

[**DISPLAY**] [*Channel 1 Meas2* ●]
[**SCALAR**] [*Input Selection*] [*Tuned Input*]
[*Tuned Input*]
[**SOURCE ON/OFF** ●]

Before any measurement is made the unit should be normalised to remove the system delay. The amplitude will be normalized too.

[**CAL**] [*Through Cal*] [*Continue*] to normalize the amplitude response
[**SELECT MEAS**]
[**CAL**] [*Through Cal*] [*Continue*] to normalize the group delay

Now insert the device under test between the attenuator

pad and the spectrum analyzer input.

    Use **[SCALE/FORMAT]** [*Set Scale*] [10] **[Gn]**
[*Set Ref Position*] **[⇩][⇩][⇩][⇩]** **[ENTRY OFF]** to move the reference position.

We can now use the instrument markers to display the 3 dB bandwidth of the filter and a number of passband parameters. Firstly to measure the 3 dB bandwidth:-

    [SELECT MEAS]
    [*Active Marker To Maximum*] [*Mkr Function*]
    [*Bandwidth*] [*Set ndB Value*] should default to -3 dB
    [*Bandwidth Search*] [*Remove Result Window*]
    [MARKER] [*More*] [*Mkr Table* ●]

The display will now look like Figure 2 with the frequency response displayed at the top and the group delay beneath. To display the group delay parameters that fall between the passband of the filter we firstly define the passband as a sub-range:

    [SELECT MEAS] to make group delay the active measurement
    [*Return To Prior Menu*] [*Pk-Pk*] [*Use Sub Range* ●]
    [*Set Sub Range Start*] enter the lower frequency value from the marker table.
    [*Set Sub Range Stop*] enter the upper frequency value from the marker table. Now any measurements are taken between these two values.

1. To measure the maximum delay change in any given bandwidth. Enter the desired bandwidth using [*Set Bandwidth*] and then [*Pk-Pk delay* ●]

2. To measure the maximum slope, press [*Measure Deviation*] [*Linear*]. The marker in the measurement channel window (top left corner) now displays the maximum gradient within the passband in ns/MHz.

3. To measure the maximum 2nd derivative, press [*Measure Deviation*] [*Parabolic*] the marker in the measurement channel window (top left corner) now displays the maximum value within the passband in ns/MHz^2.
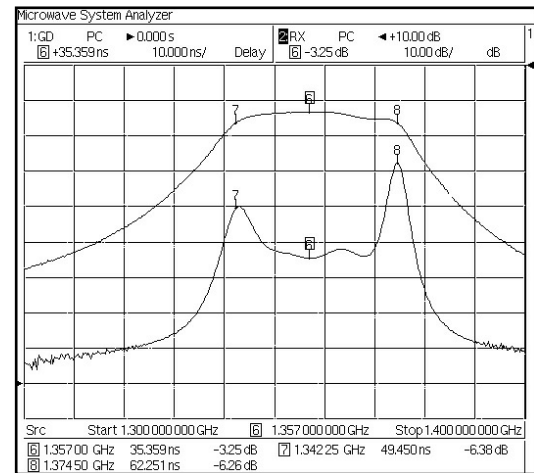

Figure 2 - Measured amplitude (top) and delay (bottom) for a 1.3 - 1.4 GHz BPF

Aperture, range and deviation

The measurement aperture is determined by the interaction between the modulation frequency and the amount of deviation. (See Appendix 2 for further information). Larger deviations result in better SNR at the FM demodulator and hence less measurement noise.

Measurement apertures of 100 kHz to 3 MHz are automatically selected, dependent upon the frequency span. Table 1 shows the makeup of the automatically selected apertures. These values were chosen for best noise performance for a given aperture.

Press **[SCALAR]** [*Input Selection*] [*Group Delay*] [*Set Aperture...*] to display the currently selected aperture.

Aperture	Range	Deviation
max 3 MHz	1 μ	1 MHz
2 MHz	1 μ	500 kHz
1 MHz	1 μ	200 kHz
500 kHz	2 μ	100 kHz
400 kHz	5 μ	100 kHz
200 kHz	5 μ	40 kHz
100 kHz	10 μ	20 kHz

Table 1 - Automatically selected ranges and deviations

The largest aperture from Table 1 is selected that falls below 3% of the frequency span, this should avoid excessive smoothing effects. If however these values are inappropriate then the aperture can be uncoupled and selected manually from the above table. For example, if 3% aperture is deemed too large, the aperture can be manually reduced.

Press **[SCALAR]** [*Input Selection*] [*Group Delay*] [*Set Aperture...*] then use the up and down arrows to change the aperture. Select **[SCALAR]** [*Input Selection*] [*Group Delay*] [*Coupled Aperture* ●] to return to the automatic settings.

Facilities are provided for the user to specify the required range and deviation. This may be useful, for example, when measuring long cables e.g. 1.5 μs. The user may select the 2 μs range to obtain an unambiguous reading, but choose the maximum deviation (1 MHz) to minimize noise. Here the

user would know that smoothing effects are not a problem since the delay of most cables is constant with frequency. To change the range and deviation directly, use **[SCALAR]** [*Input Selection*] [*Group Delay*] [*Set Range...*] and [*Set FM Deviation...*] [*Set Aperture...*] can be pressed to obtain an estimate of the resulting signal bandwidth.

If the range or deviation are changed (directly or via the aperture control), another calibration is required.

Reducing measurement noise

The amount of noise on a group delay trace depends on aperture, signal level at the spectrum analyzer connector, carrier frequency and sweep time. Larger apertures result in less noise, but can cause smoothing effects. Typical results are shown in Table 2.

Aperture	RMS noise (ns)
max 3 MHz	0.01
2 MHz	0.02
1 MHz	0.04
500 kHz	0.20
400 kHz	1.00
200 kHz	2.50
100 kHz	15.0

Table 2 - Typical variation of RMS noise with aperture (0 dBm at spectrum analyzer, 490 ms sweep time, 2 GHz carrier)

As the carrier frequency is increased, so does the group delay noise. This is due to a combination of increased phase noise and increased loss in the signal path. Typical results are shown in Table 3.

Carrier frequency	RMS noise (ns)
1 GHz	0.007
2 GHz	0.008
4 GHz	0.009
10 GHz	0.030
19.9 GHz	0.100

Table 3 - Typical variation of RMS noise with carrier frequency (0 dBm at spectrum analyzer, 10 s sweep time, 3 MHz aperture)

The sweep time control can be used to reduce the noise level. To change the sweep time, press **[SOURCE]** [*Sweep Time*] [*User Set Sweep Time*] [*Set Sweep Time...*]. When the sweep is slowed, more averaging is applied to each measurement point, resulting in reduced noise.

Video averaging can also be used to reduce measurement noise. Applying N averages will reduce the RMS noise level by √N, assuming gaussian noise. To set the number of averages, press **[SCALAR]** [*Set Average Number...*]. Video averaging allows you to see the effects of changes without waiting for one long sweep. To restart the averaging after a change, select **[SCALAR]** [*Restart Averaging*].

Noise can also be reduced using smoothing. This applies a running average across a certain percentage of the sweep. **Beware!** Excessive smoothing can cause measurement errors in the same way as a large aperture. You will notice that smoothing has no effect until certain percentages are reached. This is due to the finite number of points in the sweep - after a certain threshold, 2 points are averaged, then 3 etc. To enable smoothing, select **[SCALAR]** [*Smoothing*]

[*Smoothing* ●] [*Set Aperture...*] (turn the rotary control or enter a value).

Remember that noise is also "frozen" into the calibration, so you may need to recalibrate to benefit from increasing sweep time or averaging.

Zeroing

Group delay is derived from the phase change in the modulating signal. The system isolates phase change due to the DUT from internal phase changes due to temperature and vibration by using several internal zeroing paths. For sweeps faster than 0.5 second, the system is zeroed at the start of every sweep. For sweeps slower than 0.5 second, the system is zeroed at every measurement point. It may be necessary to recalibrate if the 0.5 second sweep time boundary is crossed.

For frequencies above 3 GHz, on the 1 μs range, the system is always zeroed at every point using a zeroing path in the microwave source. This results in greater measurement noise as two measurements are being made at each point. It is possible to disable this zeroing system by selecting **[SCALAR]** [*Input Selection*] [*Group Delay*] [*Zeroing*] [*Zeroing Off*]. This results in less noise but may result in more long term drift.

Zeroing is disabled for the 2, 5 and 10 μs ranges because, in general, system drift will not be visible under the measurement noise on these ranges. The zeroing system can be locked on for all ranges by selecting **[SCALAR]** [*Input Selection*] [*Group Delay*] [*Zeroing*] [*Zeroing On*]. This setting is appropriate when large deviations are selected on a long range for measuring long cables. Note that the measurement noise will be significantly increased.

Dynamic range

If the amplitude changes between calibration and measurement, the delay accuracy will be degraded, see Figure 3. The x-axis is the amount of loss of the device under test (DUT) and the y-axis is the group delay error. In this example, we're under 0.5 ns to about 40 dB loss.

The amplitude to delay conversion is mainly due to non-ideal behaviour of the limiting amplifier at the end of the IF strip. In most cases this won't be of concern, but care is needed when measuring (for example) lossy cables or the transition bands of filters.
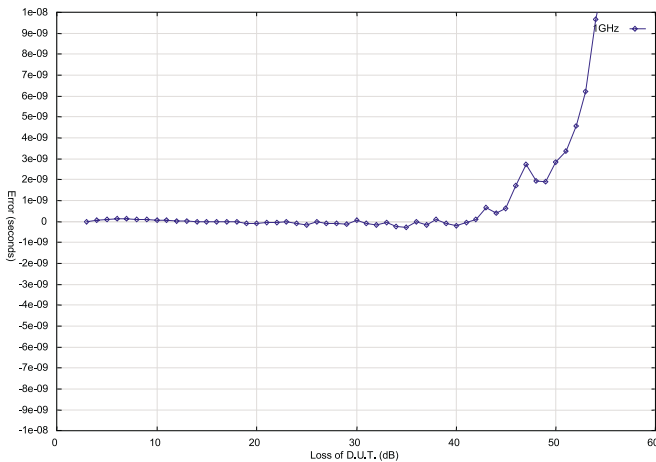
Figure 3 - Typical amplitude to delay conversion (1 µs range, 1 MHz deviation, 1 GHz carrier, +6 dBm source)

Mismatch errors

Mismatch at each end of the cables used to connect the DUT will make the measured conversion delay rippled with a sinusoidal pattern. The amplitude of the ripples can be estimated with the following formula:

$$Error = \pm 2T\rho_1\rho_2 \text{ (seconds)} \qquad \text{Equation 2}$$

where T is the length of the cable between the two mismatches, and ρ_1,ρ_2 are the *magnitudes* of the voltage reflection coefficients at the ends of the cable. The spacing of the ripples is ½T.

Example

Calculate the potential group delay error due to mismatch at the input port of a device with a 15 dB input return loss. Assume that the source return loss is 10 dB and the cable length is 10 ns.

$$Error = \pm 2 \times 10 \text{ ns} \times 10^{-\frac{15+10}{20}} = \pm 1.12 \text{ ns (seconds)}$$

Mismatch errors can be a serious effect, especially near the band edges of filters where a large proportion of the incident power is reflected.

To minimize mismatch errors use short, good quality cables and arrange the source and spectrum analyzer match to be as good as possible. The use of a 6 to 10 dB pad at the source is recommended (increase the power level to compensate if required). The spectrum analyzer match can be improved by increasing the "operating signal level" setting to activate the internal attenuator (press **[SCALAR]** [Input Selection] [Group Delay] [Set Operating Signal Level]). If problems occur, a simple error calculation as shown above may be useful.

It is important also to consider mismatch errors during the calibration by including the same padding as will be used for the measurement. Adding pads will degrade the group delay resolution as the SNR at the FM demodulator is degraded. If the mismatch ripple spacing (½T) is much greater than the bandwidth of interest, the error will be less than predicted by equation 2.

Frequency Converted Group Delay measurements

The 6840 series group delay option can be used to characterize frequency converters. No external frequency converting hardware is needed (as with most VNAs) because the source and receiver frequencies are independent. There are two basic possibilities for calibration, a through calibration or a "golden standard" calibration.
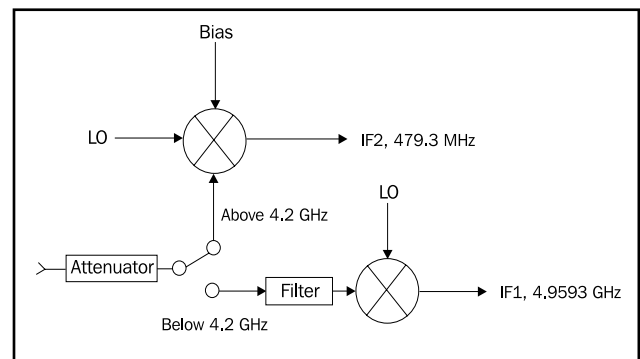
Through path calibration

Figure 4 - Simplified block diagram of 6840 spectrum analyzer front end

The 6840 source contains many delay changes due to band switching and the frequency modulation hardware. When measuring a frequency converter, this variation can be normalised by performing a through path calibration over the source frequency range. The calibration will be valid if the spectrum analyzer delay is the same over the source and receiver frequency ranges. The basic architecture of the spectrum analyzer front end is shown in Figure 4.

There are two major frequency bands, above and below 4.2 GHz. If the source and receiver frequency ranges are both in one of these bands, there will be no step due to this band change. If the source range lies in one band and the receiver in the other, there will be an offset on the measured delay but the delay flatness can still be measured. If the receiver frequency range straddles the band change, there will be a step in the measured delay.

The group delay flatness of each band is typically +/- 2 ns. If you need better accuracy than this, consider a golden standard calibration.

To set up a conversion measurement using a through path calibration, proceed as follows:

[PRESET] [Full]
[SCALAR] [Yes] [Input Selection] [Group Delay] [Group Delay]

Now set the frequency conversion parameters. This can be done using the Mixer set-up form as described in the 6840 manual or using the scale and offset parameters.

[**CAL**] [*Through Cal*] [*Source Freq. Range*]

Make the through connection

[*Continue*]

Connect the D.U.T.

Golden standard calibration

In some cases, the through path calibration described above may not give sufficient accuracy.   An alternative method uses a golden standard device with known or assumed delay performance. The instrument is calibrated using this device, which is replaced by the DUT for the measurement. This method also removes any band change steps.

A suitable golden standard device is a well matched broadband mixer with external LO - the connectorized devices from Mini-Circuits may be suitable. Another option is to use a modified DUT with the group delay critical components bypassed (usually the filters).

To set up a conversion measurement using a golden standard calibration, proceed as follows:

[**PRESET**] [*Full*]
[**SCALAR**] [*Yes*] [*Input Selection*] [*Group Delay*]
[*Group Delay*]

Now set the frequency conversion parameters. This can be done using the Mixer set-up form as described in the 6840 manual or using the scale and offset parameters.
Connect the calibration standard mixer

[**SAVE/RECALL**] [*Save Trace*] [*New Store Name*] Enter a new memory name [*Save*]

[**SAVE/RECALL**] [*Apply Trace Memory*]
[*Relative to Memory*] Select memory name [*Select*]

Connect the DUT

Example measurement

The following example shows how to set the instrument up to measure the amplitude and delay response of a 2.2 GHz to 500 MHz downconverter over a 100 MHz span. A through path calibration method is used. The internal LO frequency is 1.7 GHz (upper sideband). Proceed as follows:

[**PRESET**] [*Full*]
[**SCALAR**] [*Yes*]

Now set up the frequency conversion parameters...

[*Conversion Measurements*] [*Mixer meas set-up*]
[*Downconverter*]
[*Cntr / Span* ●] [⇩] [500] [**Mμ**] [⇩] [100] [**Mμ**] [⇩]
[*1.7*] [**Gn**]
[*Return to Conv Meas*]

Make the through connection.   Set up the two measurements and calibrate...

[**SCALAR**] [*Input Selection*] [*Group Delay*] [*Group Delay*]
[**CAL**] [*Through Cal*] [*Source Freq Range*] [*Continue*]

[**DISPLAY**] [*Channel 1 Meas 2* ●]
[**SCALAR**] [*Input Selection*] [*Tuned Input*] [*Tuned Input*]

The calibration of the group delay measurement was done at 0 dBm source power to minimize noise and present a level similar to the output level of the downconverter. We're now going to change the power level to -40 dBm to prevent the downconverter from saturating. We can do this because the source delay change with power level is small compared with the delay to be measured. If this were not the case, an attenuator should be used to reduce the source level. PC? is displayed on the group delay measurement to warn something has changed between calibration and measurement.

[**SOURCE**] [*Set Output Power*] [-40] [**ENTER**]

Set up some scales:-
[**SELECT MEAS**] to select measurement 1
[**SCALE/FORMAT**] [*Set Scale*] [10] [**Gn**]
[*Set Ref Position*] [⇩][⇩][⇩][⇩]
Now connect the downconverter. The result is shown in Figure 5.
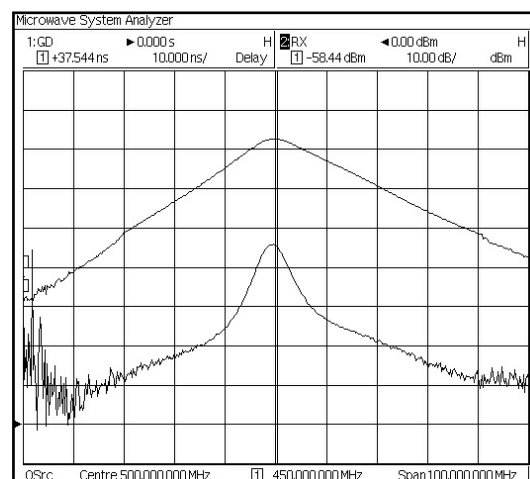

Figure 5 - Measured amplitude (top) and delay (bottom) of a downconverter (2.2 GHz to 500 MHz)

Converter LO accuracy and drift

The FM envelope delay method used by 6840 has the advantage that no access to the frequency converter LO is required (as with some VNA methods). However, there are some requirements on LO accuracy.

The 6840 spectrum analyzer is fixed to 3 MHz resolution bandwidth for group delay measurements. This, and FM demodulator considerations mean that the 6840 frequency offset must be set within +/-500 kHz of the actual frequency offset for the measurement to work.

Frequency error causes the group delay response of the spectrum analyzer filters to be traced out. This will place an offset on the trace, which won't be a problem if only flatness is of interest. If the converter LO is not very stable, this offset will drift up and down, which may be more of a problem. In this case, autoscaling may help but the only real solution is to stabilize the converter's LO. The magnitude of this effect (measured on one instrument) is 0.1 ns change per 1 kHz frequency error.

Errors due to spurious responses

For frequencies below 4.2 GHz, the spectrum analyzer front end is used as normal (up/down conversion). Sufficient filtering is provided such that spurious receiver responses are unlikely to be a problem. Above 4.2 GHz, however, the YIG preselector filter is switched out for group delay measurements. This leads to the possibility of interference from tones near images and other harmonics.

Refer to Figure 4, which shows the front end of the spectrum analyzer. Above 4.2 GHz, the first IF is 479.3 MHz. A low side LO is used. The mixer responds at N x f_{lo} ±479.3 MHz. It is most unlikely that images etc. of a converter will lie on a spurious response, indeed the situation would be much worse with a VNA which often has combs of spurious responses separated by a few hundred MHz (due to the sampling receiver). An example of a measurement that would cause problems is an unfiltered upconverting mixer with an input IF frequency close to 479.3 MHz, as the double sideband delay response would result.

APPENDIX 1 GROUP DELAY BASICS
Distortion of baseband signals

In Fourier analysis, a signal is represented as the summation of an infinite number of sinusoids. Each of these sinusoids can be written as:

$$s(t) = A\cos(\omega t + \phi)$$

We can define a signal as being undistorted if it is received exactly as sent with only an amplitude change and a delay. This requires all the sinusoids comprising the signal to be scaled and delayed by the same amount. The scaling, or amplitude requirement is intuitive, but let's look in detail at the delay. If each of the sinusoids are delayed by time T, they can each be written:

$$s(t - T) = A\cos(\omega(t - T) + \phi) = A\cos(\omega t + \phi - \omega T)$$

This is identical to the expression for the original sinusoid with a phase shift $-\omega T$. This represents a *phase shift directly proportional to frequency*. If the phase shift is not directly proportional to frequency, the signal becomes distorted. Note that constant phase shifts will also cause distortion - the phase must go through zero at $\omega=0$.

Phase delay

Consider shifting $s(t)$ by a phase angle, θ. $s(t)$ becomes:

$$A\cos(\omega t + \phi + \theta) = A\cos(\omega(t + \frac{\theta}{\omega}) + \phi) = s(t + \frac{\theta}{\omega})$$

which is the original signal delayed by -θ/ω. -θ/ω is known as phase delay, T_p. If all the components of a baseband signal undergo the same phase delay, there will be no distortion.

Distortion of modulated signals

Modulated signals can be conveniently represented by an equivalent complex baseband signal, $x(t)$. $x(t)$ can have different positive and negative frequency components, representing signals above and below the carrier frequency, ω_c. It is this signal that conveys the information, so we're interested in what conditions cause $x(t)$ to become distorted. Using lower case for time domain signals and upper case for their Fourier transform, the signal can be written:

TIME DOMAIN	FREQUENCY DOMAIN
$x(t)e^{i\omega_c t}$	$X(\omega-\omega_c)$

Now pass the signal through a device or system with frequency response $H(\omega)$, and call the result $y(t)$:

$y(t)$	$Y(\omega) = X(\omega-\omega_c)H(\omega)$

We're interested in what this process has done to our original signal, $x(t)$. We can "demodulate" the signal by multiplying $y(t)$ with a phasor of frequency ω_c.

$y(t)e^{i\omega_c t}$	$Y(\omega+\omega_c) = X(\omega+\omega_c - \omega_c)H(\omega+\omega_c)$
	$= X(\omega)H(\omega+\omega_c)$

So, it's as if our *baseband* signal, *x(t)*, has been passed through a system with the same frequency response as $H(\omega)$, but *shifted down to DC*. We'll call this response $H_{bb}(\omega)$.

Let's return to the baseband case and see why a constant phase shift, θ at all frequencies causes distortion. The sinusoidal signal, *s(t)* contains both positive and negative frequency components. This can be seen by using the Euler identity to expand the sinusoid as two exponentials:

$$A\cos(\omega t + \phi + \theta) = \frac{A}{2}(e^{j\omega t + j\phi + j\theta} + e^{-j\omega t - j\phi - j\theta})$$

The positive frequency components are shifted in phase by $+\theta$, whereas the negative frequency components are shifted in phase by $-\theta$. This effect is responsible for distortion of a baseband signal due to a constant phase shift with frequency.

For a complex baseband signal, the components are not sinuoids but exponentials, and we can introduce a constant phase shift for both positive and negative frequencies. A constant phase shift, θ results in each component having the form:

$$Ae^{j(\omega t + \phi + \theta)} = Ae^{j(\omega t + \phi)}e^{j\theta}$$

Every component has been multiplied by the same complex number, $e^{j\theta}$, so the whole signal has been scaled by a constant factor (albeit complex), hence no distortion occurs. So, the requirement on $H_{bb}(\omega)$ for distortionless transmission of complex baseband signals is relaxed to linear phase (offsets don't cause distortion). i.e. $H(\omega)$ must have flat amplitude and linear phase across the bandwidth of the modulated signal.

Group Delay

Assume the phase response of $H_{bb}(\omega)$ is given by a polynomial:

$$\phi(\omega) = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + a_4\omega^4 + a_5\omega^5 + ...$$

The coefficients are given by:

$$a_0 = \phi|_{\omega=0}$$

$$a_1 = \frac{d\phi}{d\omega}\Big|_{\omega=0}$$

$$a_2 = \frac{d^2\phi}{d\omega^2}\Big|_{\omega=0}$$

etc

<div style="text-align:right">Equation 3</div>

Let's observe the delay experienced by frequency components displaced a small amount, $\delta\omega$ from the centre. Each component is of the form:

$$s(t) = e^{j\delta\omega t}$$

After passing through $H_{bb}(\omega)$, for small $\delta\omega$ we have:

$$s_0(t) = e^{j(\delta\omega t + a_0 + a_1\delta\omega + a_2\delta\omega^2 + a_3\delta\omega^3 + a_5\delta\omega^5 + ...)}$$

$$= s(t + a_1 + a_2\delta\omega^1 + a_3\delta\omega^2 + a_4\delta\omega^3 + a_5\delta\omega^4 + ...)e^{ja_0}$$

$$\approx s(t + a_1)e^{ja_0}$$

Therefore the resulting signal has a phase shift of a_0 and a delay of $-a_1$. This delay is known as group delay and represents the delay experienced by a modulated signal ("group of frequencies") of infinitesimal bandwidth. From Equation 3, group delay is given by:

$$T_g = -\frac{\partial\phi}{\partial\omega}$$

Group delay flatness is a useful measure of phase distortion because flat group delay over a given bandwidth implies linear phase. Note that a non constant phase delay across a given bandwidth does not necessarily indicate phase distortion will occur.

To conclude, Figure 6 shows a selection of distorting and non-distorting phase responses. The signal bandwidth is shaded. It may be helpful to sketch what the phase and group delay of these responses would be.
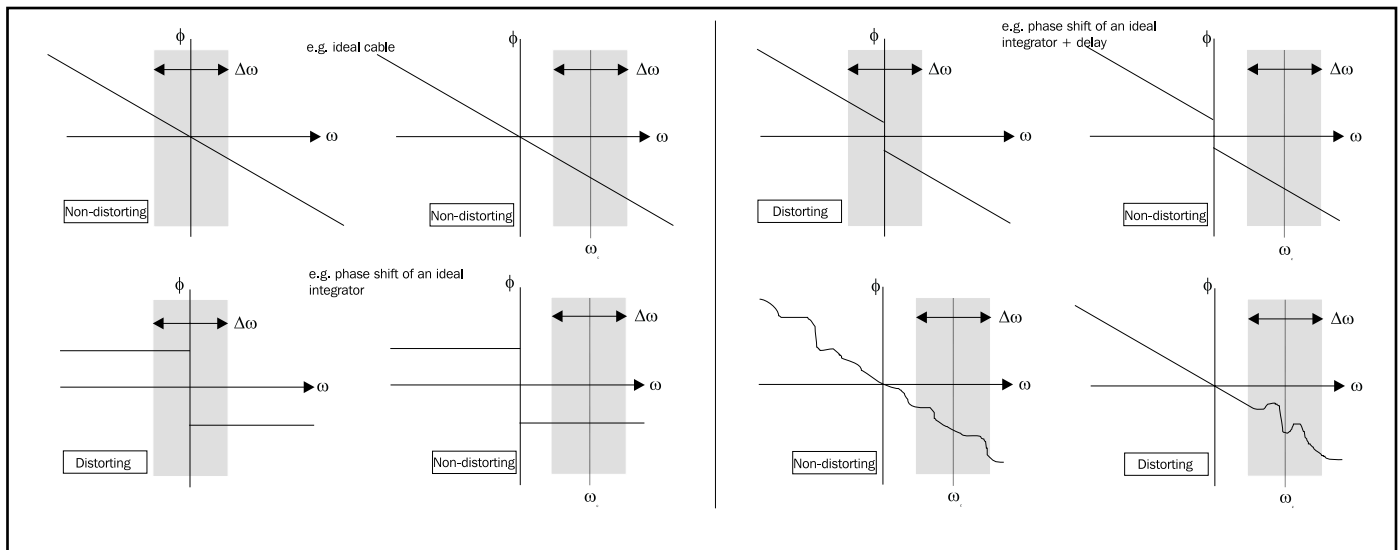

Figure 6 - Examples of distorting and non-distorting phase responses

APPENDIX 2 - MEASUREMENT TECHNIQUES

There are several techniques for measuring group delay. The most common are direct phase and envelope delay (used in 6840). Group delay can also be inferred from the amplitude response with a transform technique.

Direct phase method

If we know the phase response of the device under test (DUT), we can use equation 1 to calculate the group delay. This is the method used by a VNA, which can measure phase directly. In practice, the VNA takes a finite number of phase samples at equally spaced frequency points. Group delay is then approximated, usually with a two point derivative:-

$$T_g \approx - \frac{\phi_1 - \phi_2}{\Delta\omega}$$
Equation 4

If the group delay is not constant in the bandwidth $\Delta\omega$, the above approximation will be in error. This results in a smoothing effect, which can be minimized by reducing $\Delta\omega$. The bandwidth $\Delta\omega$ is known as the measurement aperture, which is dependent on the selected frequency span and number of points. As the aperture is reduced, evaluation of equation 44 results in greater magnification of the phase noise, and hence reduced group delay resolution. Hence, *using direct phase, reducing the measurement aperture reduces the group delay resolution*

The absolute group delay can only be determined unambiguously if the phase difference (ϕ_1-ϕ_2) is less than 180 degrees. Beyond this, we can't measure how many times the phase has wrapped. This sets a limit on the maximum absolute delay that can be measured as $\pi/\Delta\omega$. *Using direct phase, increasing the measurement aperture reduces the unambiguous absolute delay range.*

Most VNAs have reference and measurement inputs that work on the same frequency. Additional hardware (a mixer and oscillator) is usually required to perform group delay measurements on frequency converters. A major advantage of VNAs with an S parameter test set is 12 term vector error correction which allows the effects of imperfect matching to be corrected.

Transform techniques

In general smooth, "sloppy" filter responses have flatter group delay responses than those with sharp cut-offs. This observation can be expressed mathematically with the use of a Hilbert transform [1]. It is possible to use high accuracy scalar analysis to determine the amplitude response and perform a transformation to infer the group delay response.

This seemingly attractive technique has several major disadvantages. The transform relationship is only valid for linear minimum phase devices. In practice, this means devices with multiple signal paths from input to output cannot be measured (e.g. delay equalizers, SAW filters). In any case it would be difficult to prove that the device under test has a minimum phase response. In addition, the absolute delay

cannot be determined. For these reasons, transform techniques are rarely used.

Envelope delay

Group delay represents the time taken for *information* of infinitesimal bandwidth to pass through a device. Envelope delay (also known as modulation delay) measurement techniques apply information to a carrier and measure how much the information is delayed when the device is inserted. In practice, this is usually accomplished with sinusoidal modulation to control the signal bandwidth. The delay is measured with a phase detector working at the modulating frequency.

The main advantage this technique has over the direct phase implementation is that since the group delay is derived from the modulation envelope and not the carrier frequency, the technique can be applied to measure frequency converting networks. The simplified hardware also results in a cost saving.

Amplitude modulation (AM) and frequency modulation (FM) are the most commonly chosen schemes. The choice of modulation scheme depends upon the available system hardware, the DUT and the required accuracy. In a scalar network analyzer (SNA) the logical scheme would appear to be AM based, as the supplied broadband detectors will effectively demodulate amplitude-varying signals. However these detectors are diode based and the delay through the detector is dependent upon the video impedance of the diode, which changes with incident power and temperature. Therefore the detector will need to be fully delay characterized for both level and temperature [2]. Non-linear devices can cause additional problems with AM schemes, since any AM-PM conversion will introduce errors.

The constant envelope of a FM signal makes this a better choice compared to AM. A limiting amplifier preceding the demodulator can be used to remove amplitude variations. Non-linear devices can be accurately characterized (including limiting amplifiers). The FM system is also inherently less susceptible to noise. The 6840 series uses the FM envelope technique to measure group delay.

If the delay of the DUT changes over the bandwidth of the modulated signal, a smoothing effect will occur. It is therefore important to minimize the signal bandwidth. The bandwidth depends on the modulating frequency and the modulation index. As the modulation index is reduced, the SNR at the phase detector is degraded. Reducing the modulating frequency results in greater noise as each degree of phase change represents more delay. Hence, *using envelope delay, reducing the measurement aperture reduces the group delay resolution*. This is the same situation as with a VNA.

The 684x Group Delay system

The 684X MSA incorporates a source and a receiver. The group delay facility is provided with an additional module that

fits into an expansion slot inside the unit. All hooks, control and measurement software are provided internally by the MSA. Group delay is accessed via the front panel softkeys as per other measurement parameters.

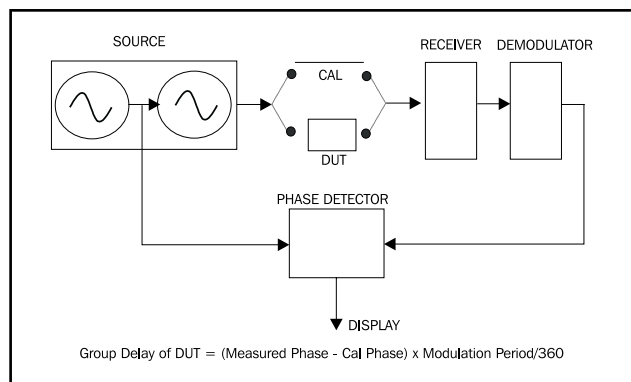Figure 7 shows a conceptual diagram of the group delay system.

Figure 7 - Conceptual diagram of 684x group delay measurement system

The group delay module contains a low frequency (LF) generator to provide the modulation source, a demodulator and a phase detector. The group delay measurement is performed by applying sinusoidal FM to the source. This is then either connected directly to the spectrum analyzer input for calibration or to the spectrum analyzer input via the DUT to make a measurement.

The spectrum analyzer converts the signal to the 10.7 MHz system IF and passes the signal to the group delay module for demodulation and phase detection. The difference in phase between the calibration and the measurement is then used to calculate the group delay.

$$T_g = \frac{\Delta\phi_{\text{degrees}}}{360 \times f_m} \qquad \text{Equation 5}$$

The phase detector can resolve phase changes of 0 to ± 180°. Beyond 180° the measurement wraps around to -180° and it is not possible to tell how many wraps have occurred. Therefore the maximum unambiguous delay is limited to:

$$T_{g\,(\text{max})} = \pm\frac{f_m}{2} \qquad \text{Equation 6}$$

Four internal modulating frequencies are provided, these are 50 kHz, 100 kHz, 250 kHz and 500 kHz and these set the maximum unambiguous delay range to ±10 µs, ±5 µs, ±2 µ and ±1 µs respectively. The higher the modulating frequency, the greater the accuracy and resolution and lower the noise. As with a VNA, increasing the aperture results in a reduced unambiguous delay range. See Appendix 3 for information about frequency modulation.

Appendix 3 - Frequency modulation

Frequency modulation is a communication technique in which the amplitude of the modulated carrier is held constant, while its frequency and rate of frequency change are controlled by the modulating signal. The frequency shift or deviation from the unmodulated carrier frequency is controlled by the amplitude of the modulating signal, whilst the rate at which the carrier shifts is controlled by the modulating signal frequency.

The frequency spectrum produced by the FM wave can be both large and complex and is governed by the interaction of the frequency deviation and the modulating signal frequency. In theory, the carrier is surrounded by an infinite set of sideband pairs spaced apart by the modulating signal frequency. The amplitude of each of the sideband pairs is dependent upon the modulation index β, which is defined as:

$$\beta = \frac{f_d}{f_m} \qquad \text{Equation 7}$$

where f_d is the peak deviation (Hertz) and f_m is the modulating frequency (Hertz). The amplitude of the Nth sideband pair relative to the unmodulated carrier amplitude is given by $J_N(\beta)$, where J_N is the Nth order Bessel function of the first kind (evaluated numerically or from tables).

In practice the amplitude of many of the sideband pairs are very small and can be ignored. This allows us to define a signal bandwidth. The f_m signal forms the measurement aperture over which the group delay is approximated.

aperture = number of significant sidebands x f_m

In the automatically selected measurement apertures, the modulated signal contains either 1, 2 or 3 sideband pairs, (all others are below 2%) which corresponds to modulation indexes of 0.4, 1 and 2. Above these modulation index values, a good approximation for the measurement aperture can be made using Carson's rule:

aperture $\approx 2(f_m + f_d)$ $\qquad$ Equation 8

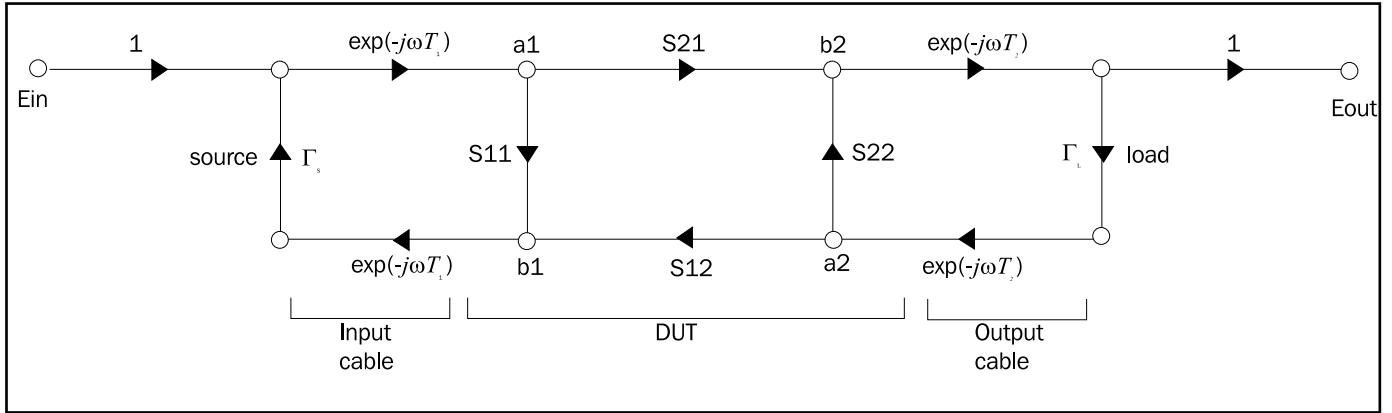## Appendix 4 - Effect of mismatch on group delay measurement

Figure 8 - Signal flow graph for mismatch analysis

In general, it is the group delay of the S_{21} parameter that is of interest. Figure 8 shows a signal flowgraph for the measurement. The cables are modelled as lossless delays, T_1 and T_2. The instrument measures Eout/Ein.

Using Mason's rules, we can write down the transmission function for this flowgraph:

$$\frac{E_{OUT}}{E_{IN}} = \frac{S_{21}e^{-j\omega(T_1+T_2)}}{1 - \Gamma_S\Gamma_L S_{21}S_{12}e^{-2j\omega(T_1+T_2)} - \Gamma_S S_{11}e^{-2j\omega T_1} - \Gamma_L S_{22}e^{-2j\omega T_2}} \qquad \text{Equation 9}$$

The numerator represents the required S_{21} parameter plus the cable delays. The denominator represents an error term, which can be drawn graphically in Figure 9.
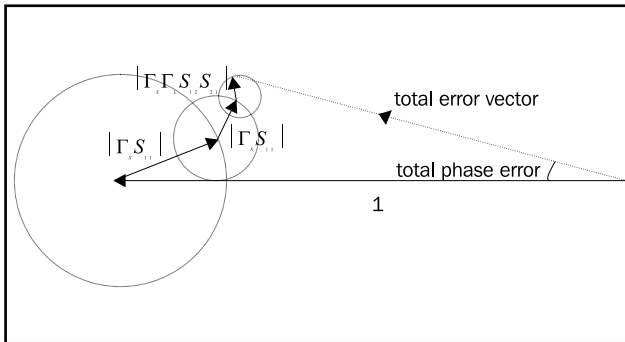

Figure 9 - Graphical representation of mismatch error

As the frequency changes, the phases of the three error phasors alter, producing a complex ripple pattern in both gain and phase. To analyze this effect further, assume the load match is very good in comparison to the source match ($|\Gamma_L| << |\Gamma_S|$). Equation 9 becomes:

$$\frac{E_{OUT}}{E_{IN}} = \frac{S_{21}e^{-j\omega(T_1+T_2)}}{1 - \Gamma_S S_{11}e^{-2j\omega T_1}} \qquad \text{Equation 10}$$

The phase of Equation 10:

$$\left\langle \frac{E_{OUT}}{E_{IN}} = \langle S_{21} - \omega(T_1+T_2) \pm \arctan\left(|\Gamma_S S_{11}|\right) \qquad \text{Equation 11}\right.$$

Assuming $\arctan(|\Gamma_S S_{11}|) = |\Gamma_S S_{11}|$ ($|\Gamma_S S_{11}|$ is small), the group delay is given by: (Equation 12)

$$T_g = \frac{\partial}{\partial\omega}\left\langle \frac{E_{OUT}}{E_{IN}} = GD_{S21} + T_1 + T_2 \pm 2T_1|\Gamma_S S_{11}| \qquad \text{Equation 12}\right.$$

The amplitude of the error ripples is $2T_1|\Gamma_S S_{11}|$. A similar argument applies when the load mismatch is much worse than the source. The period of the ripples is $1/T_1$. The analysis must be carried out for the calibration and measurement set-ups to calculate the total possible error.

References

[1] Cellai L, "A practical method to calculate group delay flatness by scalar instrumentation", Microwave Journal, February 1997

[2] Chodora J, "Group delay characterization of frequency converters", Microwave Journal, September 1995